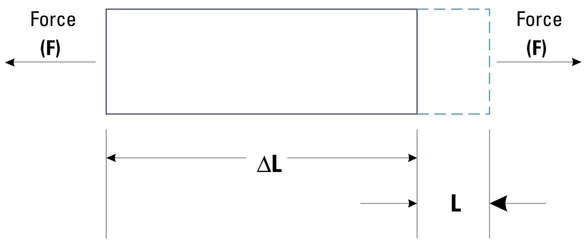


Figure 1
Uni-axial Force Applied

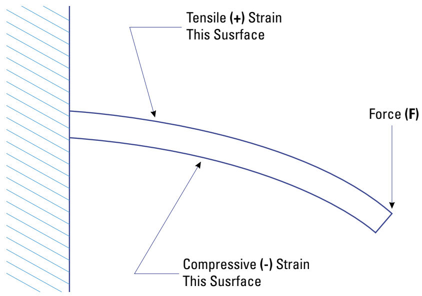


Figure 2
Cantilever in bending

Practical Strain Gage

Introduction

With today's emphasis on product liability and energy efficiency, designs must not only be lighter and stronger, but also more thoroughly tested than ever before. This places new importance on the subject of experimental stress analysis and the techniques for measuring strain. The main theme of this application note is aimed at strain measurements using bonded resistance strain gages. We will introduce considerations that affect the accuracy of this measurement and suggest procedures for improving it. We will also emphasize the practical considerations of the strain gage measurement, with an emphasis on computer controlled instrumentation.

Appendix B contains schematics of many of the ways strain gages are used in bridge circuits and the equations which apply to them.

Stress and Strain

The relationship between stress and strain is one of the most fundamental concepts from the study of mechanics of materials and is of paramount importance to the stress analyst. In experimental stress analysis we apply a given load and then measure the strain on individual members of a structure or machine. Then we use the stress strain relationships to compute the stresses in those members to verify that these stresses remain within the allowable limits for the particular materials used.

Strain

When a force is applied to a body, the body deforms. In the general case this deformation is called strain. In this application note we will be more specific and define the term STRAIN to mean deformation per unit length or fractional change in length and give it the symbol, ε. See Figure 1.

This is the strain that we typically measure with a bonded resistance strain gage. Strain may be either tensile (positive) or compressive (negative). See Figure 2. When written in equation form $\varepsilon = \Delta L/L$, we see that strain is a ratio and, therefore, dimensionless. To maintain the physical significance of strain, it is often written with units of inch/inch. For most metals the strains measured in experimental work are typically less than 0.005000 inch/inch.

Since practical strain values are so small, they are often expressed in microstrain which is $\varepsilon \times 10^{-6}$ (note this is equivalent to parts per million or ppm) and is expressed by the symbol $\mu\varepsilon$. Still another way to express strain is as percent strain, which is $\varepsilon \times 100$. For example: 0.005 inch/inch = 5000 $\mu\varepsilon$ = 0.5%.

As described to this point, strain is fractional change in length and is directly measurable. Strain of this type is also often referred to as normal strain.

Shearing Strain

Another type of strain, called SHEARING STRAIN is a measure of angular distortion. Shearing strain is also directly measurable, but not as easily as normal strain. If we had a thick book sitting on a table top and we applied a force parallel to the covers, we could see the shear strain by observing the edges of the pages. See Figure 3. Shearing strain, γ, is defined as the angular change in radians between two line segments that were orthogonal in the undeformed state. Since this angle is very small for most metals, shearing strain is approximated by the tangent of the angle

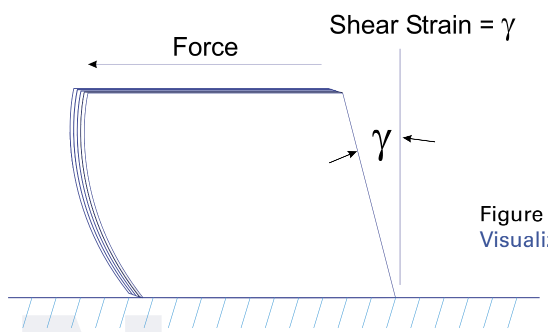


Figure 3
Visualizing Shearing Strain

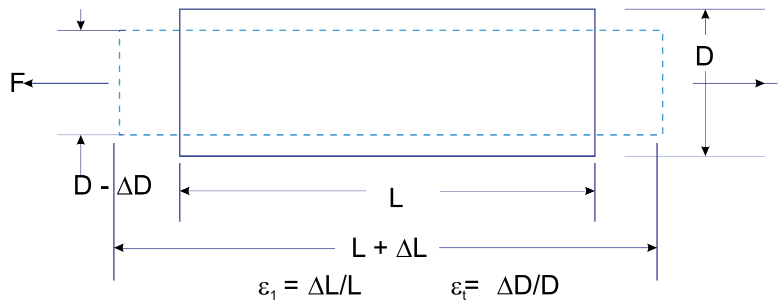


Figure 4
Poisson Strain

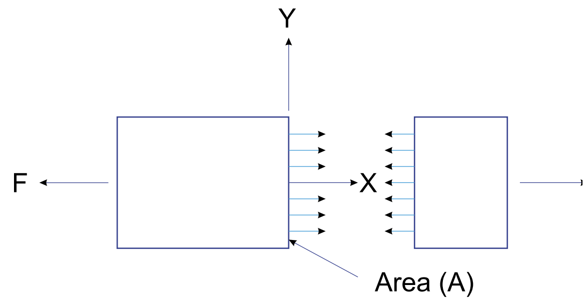


Figure 5
Normal Stress

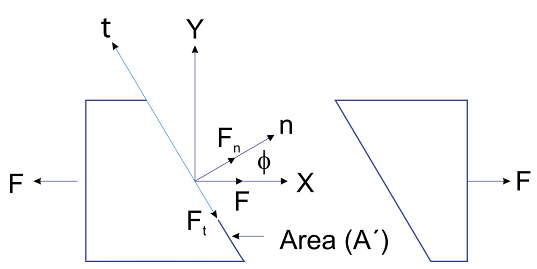
Poisson Strain

In Figure 4, a bar is shown with a uniaxial tensile force applied, like the bar in Figure 1. The dashed lines show the shape of the bar after deformation, pointing out another phenomenon, that of *Poisson strain*. The dashed lines indicate that the bar not only elongates, but that its girth contracts. This contraction is a strain in the transverse direction due to a property of the material known as Poisson's ratio. Poisson's ratio, υ, is defined as the negative ratio of the strain in the transverse direction to the strain in the longitudinal direction. It is interesting to note that no stress is associated with the Poisson strain. Referring to Figure 4, the equation for Poisson's Ratio is $\upsilon = - \varepsilon_t / \varepsilon_1$. Note that υ is dimensionless

Normal Stress

While forces and strains are measurable quantities used by the designer and stress analyst, stress is the term used to compare the loading applied to a material with its ability to carry the load. Since it is usually desirable to keep machines and structures as small and light as possible, the parts should be stressed, in service, to the highest permissible level. *STRESS* refers to force per unit area on a given plane within a body. The bar in Figure 5 has a uniaxial tensile force, F, applied along the x-axis. If we assume the force to be uniformly distributed over the cross-sectional area, A, the average

Figure 6
Shear Stress



stress on the plane of the section is F/A. This stress is perpendicular to the plane and is called *NORMAL STRESS*, σ expressed in equation form, $\sigma = F/A$. and has units of force per unit area. Since the normal stress is in the x-direction and there is no component of force in the y-direction, there is no normal Stress in that direction. The normal stress is in the positive x-direction and is tensile.

Shear Stress

Just as there are two types of strains, there is also a second type of stress called *SHEAR STRESS*. Where normal stress is normal to the designated plane, shear stress is parallel to the plane and is expressed by the symbol, τ. In the example shown in Figure 5, there is no y-component of force, therefore, no force parallel to the plane of the section, so there is no shear stress on that plane. Since the orientation of the plane is arbitrary, what happens if the plane is oriented other than normal to the line of action of the applied force? Figure 6 demonstrates this concept with a section taken on the n-t coordinate system at some arbitrary angle, ϕ, to the line of action of the force.

We see that the force vector, F, can be broken into two components, F_n and F_t, that are normal and parallel to the plane of the section. This plane has a cross-sectional area of A' and has both normal and shear stresses applied. The average normal stress, σ, is in the n-direction and the average shear stress, τ is in the t-direction. Their equations are: $\sigma = F_n/A'$ and $\tau = F_t/A'$. Note that it was the force vector that was broken into components, not the stresses, and that the resulting stresses are a function of the orientation of the section. This means that stresses (and strains), while having both magnitude and direction, are not vectors and do not follow the laws of vector addition, except in certain special cases, and they should not be treated as such. We should also note that stresses are derived quantities, computed from other measurable quantities, and are not directly measurable.

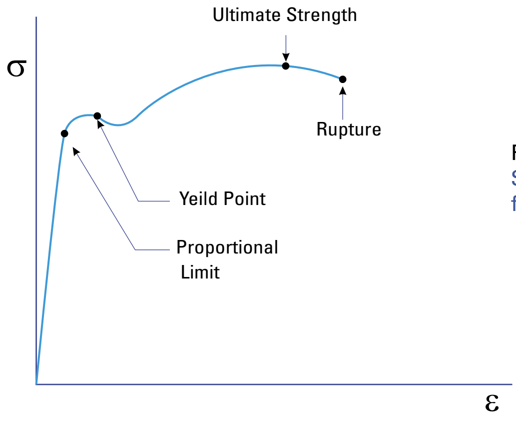


Figure 7
Stress - Strain diagram for mild steel

Practical Strain Gage

Principal Axes

In the preceding examples the x-y axes are also the PRINCIPAL AXES for the uniaxially loaded bar. By definition, the principal axes are the axes of maximum and minimum normal stress. They have the additional characteristic of zero shear stress on the planes that lie along these axes. In Figure 5 the stress in the x-direction is the maximum normal stress, and we noted that there was no force component in the y-direction and therefore zero shear stress on the plane. Since there is no force in the y-direction, there is zero normal stress in the y-direction, and in this case zero is the minimum normal stress. So the requirements for the principal axes are met by the x-y axes. In Figure 6 the x-y axes are the principal axes since that bar is also loaded uniaxially. The n-t axes in Figure 6 do not meet the zero shear stress requirement of the principal axes. The corresponding *STRAINS* on the principal axes are also maximum and minimum and the shear strain is zero.

The principal axes are very important in stress analysis because the magnitudes of the maximum and minimum normal stresses are usually the quantity of interest. Once the principal stresses are known, the normal and shear stresses in any orientation may be computed. If the orientation of the principal axes is known, through knowledge of the loading conditions or experimental techniques, the task of measuring the strains and computing the stresses is greatly simplified.

In some cases we are interested in the average value of stress or load on a member, but often we want to determine the magnitude of the stresses at a point. The material will fail at the point where the stress exceeds the load-carrying capacity of the material. This failure may occur because of excessive tensile or compressive normal stress or excessive shearing stress. In actual structures, the area of this excessive stress level may be quite small. The usual method of diagramming the stress at a point is to use an infinitesimal element that surrounds the point of interest. The stresses are then a function of the orientation of this element and, in one particular orientation, the element will have its sides parallel to the principal axes. This is the orientation that gives the maximum and minimum normal stresses on the point of interest.

Stress-Strain Relationships

Now that we have defined stress and strain, we need to explore the stress-strain relationship. It is this relationship that allows us to calculate stresses from the measured strains. If we have a bar made of mild steel and incrementally load it in uniaxial tension and plot the strain versus the normal stress in the direction of the applied load, the plot will look like the stress-strain diagram in Figure 7.

From Figure 7 we can see that, up to a point called the proportional limit, there is a linear relationship between stress and strain. Hooke's Law describes this relationship. The slope of this straight line portion of the stress-strain diagram is the *MODULUS OF ELASTICITY* or *YOUNG'S MODULUS* for the material. The modulus of elasticity, E, has the same units as stress (force per unit area) and is determined experimentally for materials. Written in equation form this stress-strain relationship is $\sigma = E * \varepsilon$. Some materials, for example, cast iron and concrete, do not have a linear portion to their stress-strain diagrams. To do accurate stress analysis studies for these materials it is necessary to determine the stress-strain properties, including Poisson's ratio, for the particular material on a testing machine. Also, the modulus of elasticity may vary with temperature. This variation may need to be experimentally determined and considered when performing stress analysis at temperature extremes. There are two other points of interest on the stress-strain diagram in Figure 7, the yield point and the ultimate strength value of stress. The yield point is the stress level at which strain will begin to increase rapidly with little or no increase in stress. If the material is stressed beyond the yield point, and then the stress is removed, the material will not return to its original size but will retain a residual offset or strain.

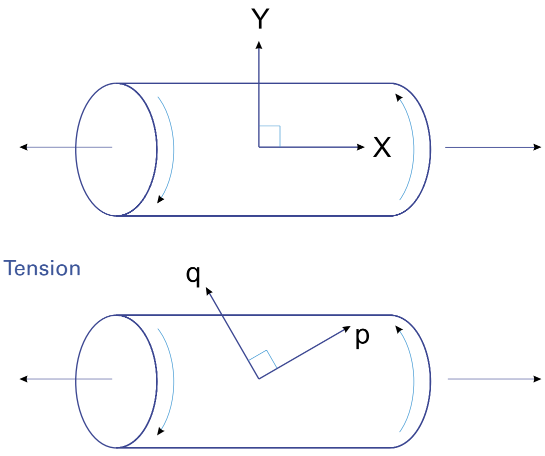


Figure 8
Shaft in Torsion and Tension

The ultimate strength is the maximum stress developed in the material before rupture.

The examples we have examined to this point have been examples of uniaxial forces and stresses. In experimental stress analysis the biaxial stress state is the most common. Figure 8 shows an example of a shaft with both tension and torsion applied. The point of interest is surrounded by an infinitesimal element, Figure 8 with its sides oriented parallel to the x-y axes. The point has a biaxial stress state and a triaxial strain state (remember Poisson's ratio). The element, rotated to be aligned with the principal (p-q) axes, is also shown in Figure 8. Figure 9 shows the element removed with arrows added to depict the stresses at the point for both orientations of the element. We see that the element oriented along the x-y axes has a normal stress in the x-direction, zero normal stress in the y-direction, and shear stresses on its surfaces. The element rotated to the p-q axes orientation has normal stress in both directions but zero shear stress as it should, by definition, if the p-q axes are the principal axes. The normal stresses, σ_p and σ_q, are the maximum and minimum normal stresses for the point. The strains in the p-q direction are also the maximum and minimum, and there is zero shear strain along these axes. Appendix C gives the equations relating stress to strain for the biaxial stress state. If we know the orientation of the principal axes, we can then measure the strain in those directions and compute the maximum and minimum normal stresses and the maximum shear stress for a given loading condition. We don't always know the orientation of the principal axes, but if we measure the strain in three separate directions, we compute the strain in any direction including the principal axes directions. Three and four element rosette strain gages are used to measure the strain when the principal axes orientation is unknown. The equations for computing the orientation and magnitudes of the principal strains from 3-element rosette strain data are found in Appendix C.

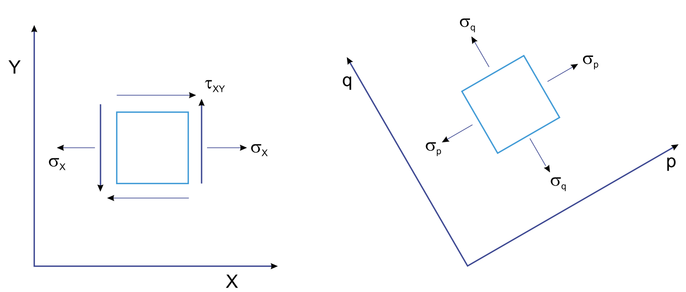


Figure 9
Element on X-Y Axes and Principal Axes

Measuring Strain

Stress in a material can't be measured directly. It must be computed from other measurable parameters. Therefore, the stress analyst uses measured strains in conjunction with other properties of the material to calculate the stresses for a given loading condition. There are methods of measuring strain or deformation based on various mechanical, optical, acoustical, pneumatic, and electrical phenomena. This section briefly describes several of the more common methods and their relative merits.

Gage Length

The measurement of strain is the measurement of the displacement between two points some distance apart. This distance is the GAGE LENGTH and is an important comparison between various strain measurement techniques. Gage length could also be described as the distance over which the strain is averaged. For example, we could (on some simple structure such as the part in Figure 10) measure the part length with a micrometer both before and during loading. Then we would subtract the two readings to get the total deformation of the part. Dividing this total deformation by the original length would yield an average value of strain for the entire part.

The gage length would be the original length of the part. If we used this technique on the part in Figure 10, the strain in the reduced width region of the part would be locally higher than the measured value because of the reduced cross-sectional area carrying the load. The

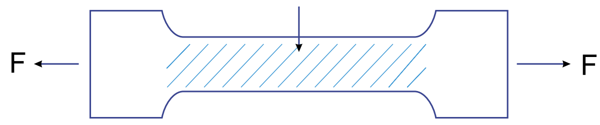


Figure 10
Area of High Local Strain and Stress

Practical Strain Gage

stresses will also be highest in the narrow region and the part will yield there before the measured average strain value indicates a magnitude of stress greater than the yield point of the material.

Ideally, we want the strain measuring device to have an infinitesimal gage length so we can measure strain at a point. If we had this ideal strain gage we would place it in the narrow portion of the specimen in Figure 10 to measure the high local strain in that region. Other desirable characteristics for this ideal strain measuring device are small size and mass, easy attachment, high sensitivity to strain, low cost, and low sensitivity to temperature and other ambient conditions.

Mechanical Devices

The earliest strain measurement devices were mechanical in nature. We have already considered an example using a micrometer to measure strain and observed a problem with that approach. Extensometers are a class of mechanical devices used for measuring strain that employ a system of levers to amplify the minute strains to a level that can be read. A minimum gage length of 1/2 inch and a resolution of about 10 $\mu\varepsilon$ is the best that can be achieved with purely mechanical devices. The addition of light beam and mirror arrangements to extensometers improves resolution and shortens gage length, allowing 2 $\mu\varepsilon$ resolution and gage lengths down to 1/4 inch.

Still another type of device, the photoelectric gage, uses a combination of mechanical, optical, and electrical amplification to measure strain. This is done by using a light beam, two fine gratings, and a photocell detector to generate an electrical current that is proportional to strain. This device comes in gage lengths as short as 1/16 inch but it is costly and delicate. All of these mechanical devices tend to be bulky and cumbersome to use, and most are only suitable for static strain measurements.

Optical Methods

Several optical methods are used for strain measurement. One of these techniques uses the interference fringes produced by optical flats to measure strain. This device is sensitive and accurate but the technique is so delicate that laboratory conditions are required for its use.

Brittle Coatings

Brittle coating techniques are another way to indicate static strain. They are often used in conjunction with strain gages. The test object is coated with a brittle lacquer and the load is applied in increments, when possible. The lacquer will crack first in the region of highest surface strain and the cracks will be perpendicular to the highest tensile strain. A representation of the full strain field is obtained indicating where to locate the strain gages and in what orientation. Under favorable conditions a reasonable estimate of the magnitude of the strain can be obtained. Getting goad data from brittle lacquer coatings is as much art as science. For high-temperature applications, a brittle ceramic coating may be used instead of the lacquer.

Electrical Devices

Another class of strain measuring devices depends on electrical characteristics which vary in proportion to the strain in the body to which the device is attached. Capacitance and inductance strain gages have been constructed but sensitivity to vibration, mounting difficulties, and complex circuit requirements keep them from being very practical for stress analysis work. These devices are, however, often employed in transducers. The piezoelectric effect of certain crystals has also been used to measure strain. When a crystal strain gage is deformed or strained, a voltage difference is developed across the face of the crystal. This voltage difference is proportional to the strain and is of a relatively high magnitude. Crystal strain gages are fairly bulky, very fragile, and not suitable for measuring static strains.

Probably the most important electrical characteristic, which varies in proportion to strain, is that of electrical resistance. Devices whose output depend on this characteristic are the piezoresistive or semiconductor gage, the carbon-resistor gage, and the bonded metallic wire and foil resistance gages. The carbon resistor gage is the forerunner of the bonded resistance wire strain gage. It is low in cost, can have a short gage length, and is very sensitive to strain. A high sensitivity to temperature and humidity are the disadvantages of the carbon-resistor strain gage.

The semiconductor strain gage is based on the piezoresistive effect in certain semiconductor materials such as silicon and germanium. Semiconductor gages have elastic behavior and can be produced to have either positive or negative resistance changes when strained. They can be made physically small while still maintaining a high nominal resistance. The strain limit for these gages is in the 1000 to 10000 με range with most tested to 3000 με in tension. Semiconductor gages exhibit a high sensitivity to strain but the change in resistance with strain is nonlinear. Their resistance and output are temperature sensitive and the high output, resulting from changes in resistance as large as 10-20%, can cause measurement problems when using the devices in a bridge circuit. However, mathematical corrections for the temperature sensitivity, the nonlinearity of output, and the nonlinear characteristics of the bridge circuit (if used), can be made automatically when using computer controlled instrumentation to measure strain with semiconductor gages. They can be used to measure both static and dynamic strains. When measuring dynamic strains, temperature effects are usually less important than for static strain measurements, and the high output of the semiconductor gage is an asset.

The bonded resistance strain gage is by far the most widely used strain measurement tool for today's experimental stress analyst. It consists of a grid of very fine wire, or more recently, of thin metallic foil bonded to a thin insulating backing called a carrier matrix. The electrical resistance of this grid material varies linearly with strain. In use, the carrier matrix is attached to the test specimen with an adhesive. When the specimen is loaded, the strain on its surface is transmitted to the grid material by the adhesive and carrier system. The strain in the specimen is found by measuring the change in the electrical resistance of the grid material. The bonded resistance strain gage is low in cost, can be made with a short gage length, is only moderately affected by temperature changes, has small physical size and low mass, and has fairly high sensitivity to strain. It is suitable for measuring both static and dynamic strains. The remainder of this application note deals with the instrumentation considerations for making accurate, practical strain measurements using the bonded resistance strain gage.

The Bonded Resistance Strain Gage

The term "bonded resistance strain gage" can apply to the nonmetallic (semiconductor) gage or to the metallic (wire or foil) gage. Wire and foil gages operate on the same basic principles and both can be treated in the same fashion from the measurement standpoint. The semiconductor gage, having a much higher sensitivity to strain than metallic gages, can have other considerations introduced into its measurement. We will use the term STRAIN GAGE or GAGE to refer to the BONDED METALLIC FOIL GRID RESISTANCE STRAIN GAGE throughout the rest of this application note. These foil gages are sometimes referred to as metal film gages.

Strain gages are made with a printed circuit process using conductive alloys rolled to a thin foil. The alloys are processed, including controlled atmosphere heat treating, to optimize their mechanical properties and temperature coefficient of resistance. A grid configuration for the strain sensitive element is used to allow higher values of gage resistance while maintaining short gage lengths. Gage resistance values range from (30 to 3000) Ω, with 120 Ω and 350 Ω being the most commonly used values for stress analysis. Gage lengths from 0.008 inch to 4 inches are commercially available. The conductor in a foil grid gage has a large surface area for a given cross-sectional area. This keeps the shear stress low in the adhesive and carrier matrix as the strain is transmitted by them. This larger surface area also allows good heat transfer between grid and specimen. Strain gages are small and light, will operate over a wide temperature range, and can respond to both static and dynamic strains. They have wide application and acceptance in transducers as well as stress analysis.

In a strain gage application, the carrier matrix and the adhesive must work together to faithfully transmit the strains from the specimen to the grid. They also serve as an electrical insulator between the grid and the specimen and must transfer heat away from the grid. Three primary factors influencing gage selection are operating temperature, state of strain (including gradients, magnitude and time dependence), and stability requirements for the gage installation. The importance of selecting the proper combination of carrier material, grid alloy, adhesive, and protective coating for the given application cannot be over-emphasized. Strain gage manufacturers are the best source of information on this topic and have many excellent publications to assist the customer in selecting the proper strain gages, adhesives, and protective coatings.

Gage Factor

When a metallic conductor is strained, it undergoes a change in electrical resistance, and it is this change that makes the strain gage a useful device. The measure of this resistance change with strain is GAGE FACTOR,

Practical Strain Gage

GF. Gage factor is defined as the ratio of the fractional change in resistance to the fractional change in length (strain) along the axis of the gage. Gage factor is a dimensionless quantity and the larger the value, the more sensitive the strain gage. Gage factor is expressed in equation form as:

$$GF = \frac{\Delta R / R}{\Delta L / L} = \frac{\Delta R / R}{\varepsilon}$$

Equation 1

It should be noted that the change in resistance with strain is not just due to the dimensional changes in the conductor, but that the resistivity of the conductor material also changes with strain. The term gage factor applies to the strain gage as a whole, complete with carrier matrix, not just to the strain sensitive conductor. The gage factor for constantan and nickel-chromium alloy strain gages is nominally 2 and various gage and instrumentation specifications are usually based on this nominal value.

Transverse Sensitivity

If the strain gage were a single straight length of conductor, and of small diameter with respect to its length, it would respond to strain along its longitudinal axis and be essentially insensitive to strain perpendicular or transverse to this axis. For any reasonable value of gage resistance, it would also have a very long gage length. When the conductor is in the form of a grid to reduce the effective gage length, there are small amounts of strain sensitive material in the end loops or turnarounds that lie transverse to the gage axis. This end loop material gives the gage a non-zero sensitivity to strain in the transverse direction. TRANSVERSE SENSITIVITY FACTOR, K_t is defined by

$$K_t = \frac{GF\,(transverse)}{GF\,(longitudinal)}$$

and is usually expressed in percent. Values of K_t range from 0 to 10%. To minimize this effect, extra material is added to the conductor in the end loops and the grid lines are kept close together. This serves to minimize the resistance in the transverse direction. Correction for transverse sensitivity may be necessary for short, wide grid gages, or where there is considerable misalignment between the gage axis and the principal axis, or in rosette analysis where high transverse strain fields may exist. Data supplied by the manufacturer with the gage can be entered into the computer controlling the instrumentation and corrections for transverse sensitivity made to the strain data as it is collected.

Temperature Effects

Ideally we would prefer the strain gage to change resistance only in response to the stress induced strain in the test specimen, but the resistivity and the strain sensitivity of all known strain sensitive materials vary with temperature. Of course that means the gage resistance and the gage factor will change when the temperature changes. This change in resistance with temperature for a mounted strain gage is a function of the difference in the thermal expansion coefficients between the gage and the specimen and of the thermal coefficient of resistance of the gage alloy. Self-temperature compensating gages may be produced for specific materials by processing the strain sensitive alloy such that it has thermal resistance characteristics that compensate for the effects of the mismatch in thermal expansion coefficients between the gage and the specific material. A temperature compensated gage produced in this manner is accurately compensated only when mounted on a material that has a specific coefficient of thermal expansion. Table 2 is a list of common materials for which self temperature compensated gages are available.

Material	PPM/˚C
Quartz	0.5
Titanium	9
Mild Steel	11
Stainless Steel	16
Aluminum	23
Magnesium	26

Table 2
Thermal expansion coefficients of some common materials for which temperature compensated strain gages are available

The compensation is only effective over a limited temperature range because of the nonlinear character of both the thermal coefficient of expansion and the thermal coefficient of resistance. The gage manufacturer supplies information specifying the accuracy of the temperature compensation in the form of an APPARENT STRAIN curve. This is a plot of temperature-induced apparent strain versus temperature, for the gage, mounted on a specific material with a specified coefficient of thermal expansion. The equation for this curve can be obtained from the gage manufacturer by applying curve fitting techniques to the graph supplied with the gage, or by generating an apparent strain curve with the actual gage after installation.

If we monitor the temperature at the gage during the strain measurement we can solve this equation to compensate for temperature-induced apparent strain. A later section of this application note discusses

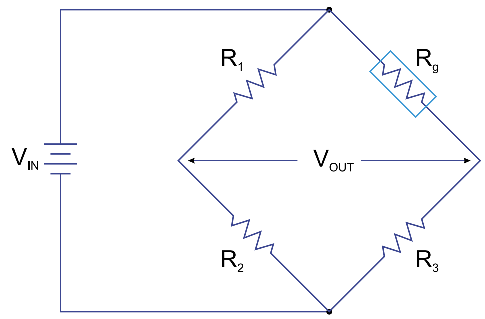


Figure 11
Wheatstone Bridge Circuit

Technical Note

temperature compensation using this technique and also a correction method for using a temperature compensated gage on a material with a thermal coefficient of expansion different from that used by the manufacturer for the apparent strain curve. The manufacturer also supplies data, usually in the form of a graph, that shows how gage factor varies with temperature, so that the strain data can also be corrected for this temperature effect.

The Measurement

From the gage factor equation we see that it is the FRACTIONAL CHANGE in resistance that is the important quantity rather than the absolute resistance value of the gage. Let's see just how large this resistance change will be for a strain of 1 με. If we use a 120 Ω strain gage with a gage factor of +2, the gage factor equation tells us that 1 με applied to a 120 Ω gage produces a change in resistance of

$\Delta R = 120 \times 0.000001 \times 2 = 0.000240\ \Omega$,

or 240 μΩ. That means we need to have microohm sensitivity in the measuring instrumentation. Since it is the fractional change in resistance that is of interest and since this change will likely be only tens of milliohms, some reference point is needed from which to begin the measurement. The nominal value of gage resistance has a tolerance equivalent to several hundred microstrain and will usually change when the gage is bonded to the specimen, so this nominal value can't be used as a reference.

An initial, unstrained, gage resistance is used as the reference from which strain is measured. Typically, the gage is mounted on the test specimen and wired to the instrumentation while the specimen is maintained in an unstrained state. A reading taken under these conditions is the unstrained reference value and applying a strain to the specimen will result in a resistance change from this value. If we had an ohmmeter that was accurate and sensitive enough to make the measurement, we would measure the unstrained gage resistance and then subtract this unstrained value from the subsequent strained values. Dividing the result by the unstrained value would give us the fractional resistance change caused by strain in the specimen. In some cases it is practical to use just this method, and these cases will be discussed in a later section of this application note. A more sensitive way of measuring small changes in resistance is with the use of the Wheatstone bridge circuit, and in fact, most instrumentation for measuring static strain uses this circuit.

Measurement Methods

Wheatstone Bridge Circuit

Because of its outstanding sensitivity, the Wheatstone bridge circuit (depicted in Figure 11) is the most frequently used circuit for static strain measurements. This section examines this circuit and its application to strain gage measurements. By using a computer in conjunction with the measurement instrumentation, we can simplify using the bridge circuit, increase measurement accuracy, and compile large quantities of data from multi-channel systems. The computer also removes the requirement for balancing the bridge, compensates for nonlinearities in output, and handles the switching and data storage in multi-channel applications.

Balanced Bridge Strain Gage Measurement

In Figure 11, V_{IN} is the input voltage to the bridge, R_g is the resistance of the strain gage, R_1, R_2 and R_3 are the resistances of the bridge completion resistors, and V_{OUT} is the bridge output voltage. A 1/4-bridge configuration exists when one arm of the bridge is an active gage and the other arms are fixed value resistors or unstrained gages, as is the case in this circuit. Ideally, the strain gage, R_g, is the only resistor in the circuit that varies, and then only due to a change in strain on the surface of the specimen to which it is attached. V_{OUT} is a function of V_{IN}, R_1, R_2. R_3 and R_g.
This relationship is:

$$Vout = Vin\left[\frac{R3}{R3 + Rg} - \frac{R2}{R1 + R2}\right]$$

Equation 2

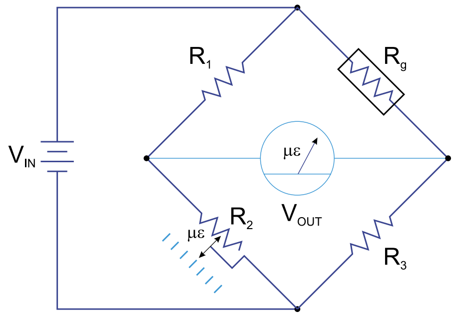


Figure 12
Bridge circuit with provision for balancing the bridge

Practical Strain Gage

When $(R_1/ R_2) = (R_g / R_3)$, V_{OUT} becomes zero and the bridge is balanced. If we could adjust one of the resistor values, R_2 for example, then we could balance the bridge for various values of the other resistors. Figure 12 shows a schematic of this concept.

Referring to the gage factor equation,

$$GF = \frac{\Delta R / R}{\varepsilon}$$

Equation 1

we see that the quantity we need to measure is the fractional change in gage resistance from the unstrained value to the strained value. If, when the gage is unstrained, we adjust R_2 until the bridge is balanced and then apply strain to the gage, the change in R_g due to the strain will unbalance the bridge and V_{OUT} will become non-zero. If we adjust the value of R_2 to once again balance the bridge, the amount of the change required in resistance R_2 will equal the change in R_g due to the strain. Some strain indicators work on this principal by incorporating provisions for inputting the gage factor of the gage being used and indicating the change in the variable resistance, R_2, directly in microstrain.

In the previous example, the bridge becomes unbalanced when the strain is applied. V_{OUT} is a measure of this imbalance and is directly related to the change in R_g, the quantity of interest. Instead of rebalancing the bridge we could install an indicator, calibrated in microstrain, that responds to V_{OUT}. Refer to Figure 12. If the resistance of this indicator is much greater than that of the strain gage, its loading effect on the bridge circuit will be negligible. i.e., negligible current will flow through the indicator. This method often assumes: 1) a linear relationship between V_{OUT} and strain, 2) a bridge that was balanced in the initial, unstrained, state, 3) a known value of V_{IN}. In a bridge circuit the relationship between V_{OUT} and strain is nonlinear, but for strains up to a few thousand microstrain, the error is usually small enough to be ignored. At large values of strain, corrections must be applied to the indicated reading to compensate for this nonlinearity.

The majority of commercial strain indicators use some form of balanced bridge for measuring resistance strain gages. In multi-channel systems the number of manual adjustments required for balanced bridge methods becomes cumbersome to the user. Multi-channel systems, under computer control, eliminate these adjustments by using an unbalanced bridge technique.

Unbalanced Bridge Strain Gage Measurement

The equation for V_{OUT} can be rewritten in the form of the ratio of V_{OUT} to V_{IN}

$$\frac{Vout}{Vin} = \left[\frac{R3}{R3 + Rg} - \frac{R2}{R1 + R2}\right]$$

Equation 3

This equation holds for both the unstrained and the strained condition. Defining the unstrained value of gage resistance as R_g and the change due to strain as ΔR_g, the strained value of gage resistance is $R_g + \Delta R_g$. The actual effective value of resistance in each bridge arm is the sum of all the resistances in that arm and may include such things as lead wires, printed circuit board traces, switch contact resistance, interconnects, etc. As long as these resistances remain unchanged between the strained and unstrained readings, the measurement will be valid. Let's define a new term, V_r, as the difference of the ratios of V_{OUT} to V_{IN} from the unstrained to the strained state:

$$Vr = \left[\left(\frac{Vout}{Vin}\right)_{strained} - \left(\frac{Vout}{Vin}\right)_{unstrained}\right]$$

Equation 4

By substituting the resistor values that correspond to the two (V_{OUT}/V_{IN}) terms into this equation, we can derive an equation for $\Delta R_g/R_g$.

This new equation is:

$$\frac{\Delta R}{Rg} = \frac{-4Vr}{1 + 2\,Vr}$$

Equation 5

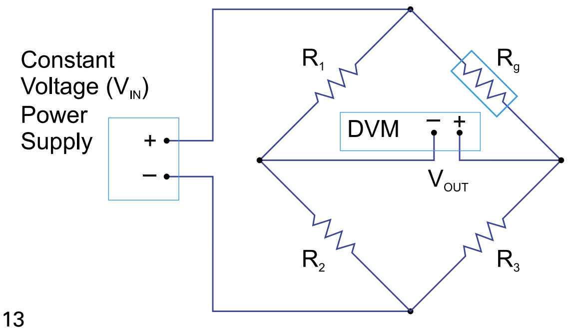


Figure 13
Instrumentation for unbalanced bridge strain gage measurement

Note that it was assumed in this derivation that ΔR_g was the only change in resistance from the unstrained to the strained condition. Recalling the equation for gage factor:

$$GF = \frac{\Delta R_g / R_g}{\varepsilon}$$

Equation 1

and combining these two equations we get an equation for strain in terms of V_r and GF

$$\varepsilon = \frac{-4V_r}{GF\left(1 + 2V_r\right)}$$

Equation 6

The schematic in Figure 13 shows how we can instrument the unbalanced bridge.

A constant voltage power supply furnishes V_{IN} and a digital voltmeter (DVM) is used to measure V_{OUT}. The DVM for this application should have a high (greater than 10^9 Ω) input resistance and 1 µV or better resolution. The gage factor is supplied by the gage manufacturer. In practice we would use a computer to have the DVM read and store V_{OUT} under unstrained conditions, then take another reading of V_{OUT} after the specimen was strained. Since the values for gage factor and the excitation voltage, V_{IN}, are known, the computer can calculate the strain value indicated by the change in bridge output voltage. If the value of V_{IN} is unknown or subject to variations with time, we can have the DVM measure it at the time V_{OUT} is measured to get a more precise value for V_r. This timely measurement of V_{IN} greatly reduces the stability requirements of the power supply, allowing a lower cost unit to be used. Note that in the preceding 1/4-bridge example, the bridge was not assumed to be balanced nor its output approximated as truly linear. Instead we derived the equation for strain in terms of quantities that are known, or can be measured, and let the computer solve the equation to obtain the exact strain value.

In the preceding example we made some assumptions that impact the accuracy of the strain measurement:

- Resistance in the three inactive bridge arms remained constant from unstrained to strained readings.
- DVM accuracy, resolution, and stability were adequate for the required measurement.
- Resistance change in the active bridge arm was due only to change in strain.
- V_{IN} and gage factor were both known quantities.

Appendix B shows schematics of several configurations of bridge circuits, using strain gages, and gives the equation for strain as a function of V_r for each.

Practical Strain Measurement

Shielding and Guarding Interference Rejection

The low output level of a strain gage makes strain measurements susceptible to interference from other sources of electrical energy. Capacitive and magnetic coupling to long cable runs, electrical leakage from the specimen through the gage backing, and differences in grounding potential are but a few of the possible sources of difficulty. The results of this type of electrical interference can range from a negligible reduction in accuracy to rendering the data invalid.

The Noise Model

In Figure 14, the shaded portion includes the Wheatstone bridge strain gage measuring circuit seen previously in Figures 12 and 13. The single active gage, R_g, shown mounted on a test specimen - e.g., an airplane tail section. The bridge excitation source V_{IN}, bridge completion resistors, R_1, R_2 and R_3. and the DVM represent the measurement equipment located a significant distance (e.g., 100 feet) from the test specimen. The strain gage is connected to the measuring equipment via three wires having resistance R_ℓ in each wire. The electrical interference which degrades the strain measurement is coupled into the bridge through a number of parasitic resistance and capacitance elements. In this context, the term “parasitic” implies that the elements are unnecessary to the measurement, are basically unwanted, and are to some extent unavoidable. The parasitic elements result

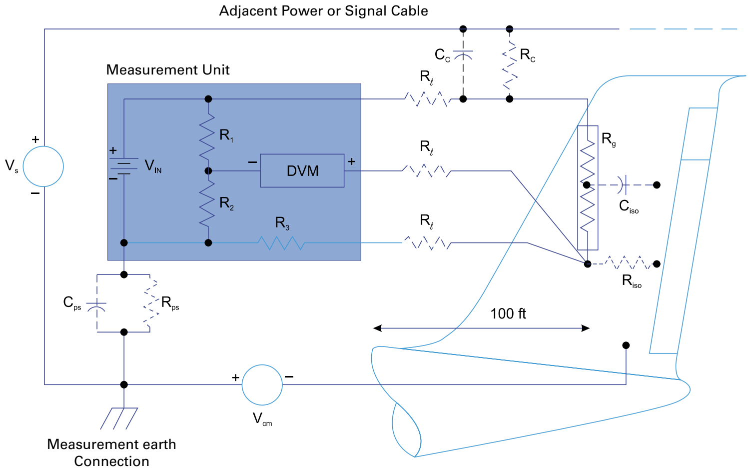


Figure 14
Remote quarter-bridge measurement illustrating parasitic elements and interference sources

Practical Strain Gage

from the fact that lead wires have capacitance to other cables, gages have capacitance to the test specimen, and gage adhesives and wire insulation are not perfect insulators - giving rise to leakage resistance. Examining the parasitic elements in more detail, the active gage, R_g, is shown made up of two equal resistors with C_{iso} connected at the center. C_{iso} represents the capacitance between the airplane tail section and the gage foil. Since the capacitance is distributed uniformly along the gage grid length, we approximate the effect as a "lumped" capacitance connected to the gage's midpoint. R_{iso} and C_{iso} determine the degree of electrical isolation from the test specimen which is often electrically grounded or maintained at some "floating" potential different than the gage. Typical values of R_{iso}, and C_{iso}, are 1000 MΩ and 100 pF, respectively. Elements C_c and R_c represent the wire-to-wire capacitance and insulation resistance between adjacent power or signal cables in a cable vault or cable bundle. Typical values for C_c and Rc are 30 pF and 10^{12} Ω per foot for dry insulated conductors in close proximity.

The power supply exciting the bridge is characterized by parasitic elements C_{ps} and R_{ps}. A line-powered, "floating output" power supply usually has no deliberate electrical connection between the negative output terminal and Earth via the third wire of its power cord. However, relatively large amounts of capacitance usually exist between the negative output terminal circuits and the chassis, and between the primary and secondary windings of the power transformer. The resistive element, R_{ps}, is due to imperfect insulators and may be reduced several decades by ionic contamination or moisture due to condensation or high ambient humidity. If the power supply does not feature floating output, R_{ps} may be a fraction of an ohm. It will be shown that use of a non-floating or grounded output power supply drastically increases the mechanisms causing electrical interference in a practical, industrial environment. Typical values for C_{ps} and R_{ps} for floating output, laboratory grade power supplies are 0.01 μF and 100 MΩ respectively. It is important to realize that neither the measuring equipment nor the gages have been grounded at any point. The entire system is floating to the extent allowed by the parasitic elements.

To analyze the sources of electrical interference we must first establish a reference potential. Safety considerations require that the power supply, DVM, bridge completion, cabinets, etc., all be connected to earth ground through the third wire of their power cords. In Figure 14, this reference potential is designated as the measurement earth connection. The test specimen is often grounded (for safety reasons) to the power system at a point some distance away from the measurement equipment. This physical separation often gives rise to different grounding potentials as represented by the voltage source, V_{cm}. In some cases, functional requirements dictate that the test specimen be "floated" or maintained many volts away from power system ground by electronic power supplies or signal sources. In either case, V_{cm} may contain constant and/or time varying components - most often at power line related frequencies.

Typical values of V_{cm}, the common mode voltage, range from millivolts due to IR drops in "clean" power systems to 250 V for specimens floating at power line potentials, for example, parts of an electric motor. The disturbing source, V_s, is shown connected to measurement earth and represents the electrical potential of some cable in close proximity, but unrelated functionally, to the gage wires. In many applications, these adjacent cables may not exist or may be so far removed as to not affect the measurement. They will be included here to make the analysis general and more complete.

Shielding of Measurement Leads

The need for using shielded measurement leads can be seen by examining the case shown in Figure 15. Here an insulation failure (perhaps due to moisture) has reduced parasitic R_c to a few thousand ohms and dc current is flowing through the gage measurement leads as a result of the source V_s. Negligible current flows through the DVM because of its high impedance. The currents through R_g and R_ℓ, develop error-producing IR drops inside the measurement loops.

In Figure 16, a shield surrounds the three measurement leads, and the current has been intercepted by the shield and routed to the point where the shield is connected to the bridge. The DVM reading error has been eliminated. Capacitive coupling from the signal

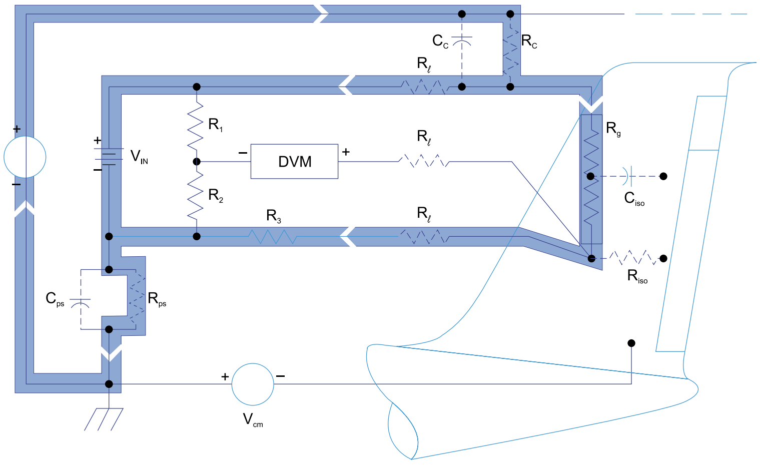


Figure 15
Current leakage from adjacent cable flows through gage wires causing measurement error

cable to unshielded measurement leads will produce similar voltage errors, even if the coupling occurs equally to all three leads. In the case where Vs is a high voltage sine wave power cable, the DVM error will be substantially reduced if the voltmeter integrates the input for a time equal to an integer number of periods (e.g., 1, 10, or 100) of the power line wave form. The exact amount of the error reduction depends upon the DVM's normal mode rejection, which can be as large as 60 dB - 140 dB or 10^3:1 - 10^7:1. If the DVM is of a type having a very short sampling period, i.e., less than 100 µs, it will measure the instantaneous value of dc signal (due to strain) plus interference. Averaging the proper number of readings can reduce the error due to power line or other periodic interference.

In the situation where the measurement leads run through areas of high magnetic fields, near high current power cables, etc., twisted measurement leads minimize the loop areas formed by the bridge arms and the DVM thereby reducing measurement degradation as a result of magnetic induction. The flat, three conductor, side-by-side, molded cable commonly used for strain gage work approaches the effectiveness of a twisted pair by minimizing the loop area between wires. The use of shielded, twisted leads and a DVM which integrates over one or more cycles of the power line waveform should be considered whenever leads are long, traverse a noisy electromagnetic environment, or when highest accuracy is required.

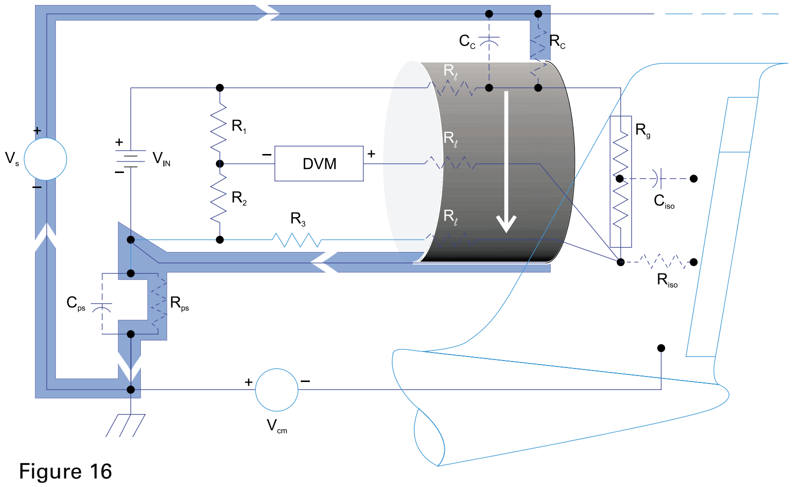


Figure 16
Addition of a metal shield around the gage wires keeps current due to Vs out of measurement leads

Guarding the Measuring Equipment

Figure 17 shows the error producing current paths due to the common mode source, V_{cm}, entering the measurement loop via the gage parasitic elements, C_{iso} and R_{iso}. In the general case, both ac and dc components must be considered. Again, current flow through gage and lead resistances results in error voltages inside the bridge arms. Tracing either loop from the DVM's negative terminal to the positive terminal will reveal unwanted voltages of the same polarity in each loop. The symmetry of the bridge structure in no way provides cancellation of the effects due to current entering at the gage.

Whereas, shielding kept error-producing currents out of the measurement loop by intercepting the current, guarding controls current flow by exploiting the fact that no current will flow through an electrical component having both of its terminals at the same potential.

In Figure 18, a guard lead has been connected between the test specimen (in close proximity to the gage) and the negative terminal of the power supply. This connection forces the floating power supply and all the measuring equipment - including the gage - to the same electrical potential as the test specimen. Since the gage and the specimen are at the same potential, no error-producing current flows through C_{iso} and R_{iso} into the measuring loops. Another way of interpreting the result is to say that the guard lead provides an alternate current path around the measuring circuit.

It should be observed that if the power supply and the rest of the measuring circuits could not float above earth or chassis potential, the guarding technique would reduce the interference by factors of only 2:1 or 4:1. Proper guarding with a floating supply should yield improvements on the order of 10^5:1 or 100 dB.

In situations where it is possible to ground the test specimen at measurement earth potential, the common mode source, V_{cm}, will be essentially eliminated.

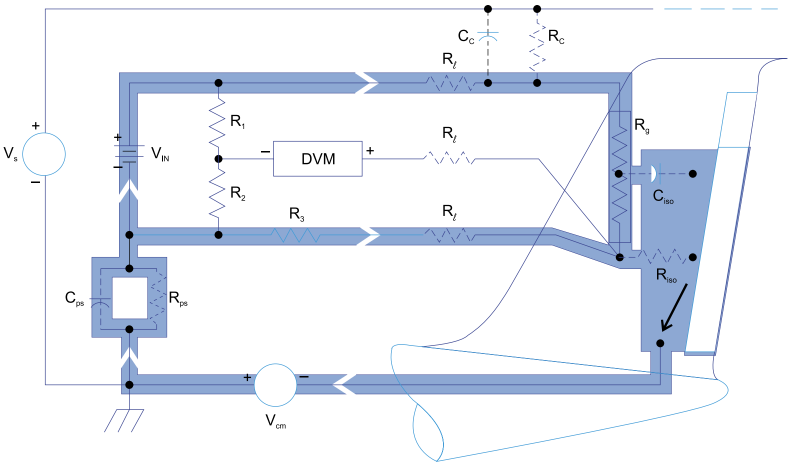


Figure 17
Error producing common mode current path

Practical Strain Gage

Bridge Excitation Level

The bridge excitation voltage level affects both the output sensitivity and the gage self heating. From the measurement standpoint, a high excitation level is desirable but a lower level reduces gage self heating. The electrical power in the gage is dissipated as heat which must be transferred from the gage to the surroundings. In order for this heat transfer to occur, the gage temperature must rise above that of the specimen and the air. The gage temperature is therefore a function of the ambient temperature and the temperature rise due to power dissipation.

An excessive gage temperature can cause various problems. The carrier and adhesive materials are no longer able to faithfully transmit the strain from the specimen to the grid if the temperature becomes too high. This adversely affects hysteresis and creep and may show up as instability under load. Zero or unstrained stability is also affected by high gage temperatures. Temperature compensated gages suffer a loss of compensation when the temperature difference between the gage grid and the specimen becomes too large, when the gage is mounted on plastics. Excessive power dissipation can elevate the temperature of the specimen under the gage to the point that the material properties of the specimen change.

The power that must be dissipated as heat by the gage in a bridge circuit with equal resistance arms is given by the following equation:

$$P = V^2 / 4R_g = (I^2)\, R_g$$

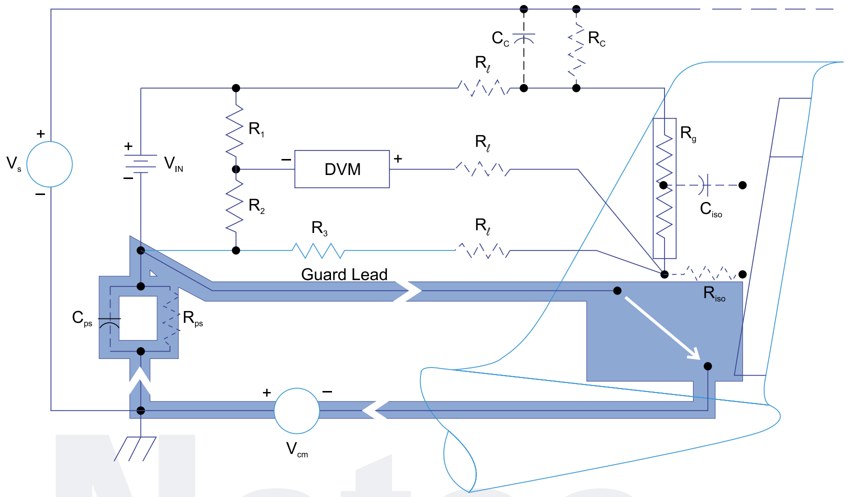
Equation 7

Where P is the power in watts, R_g is the gage resistance, I is the current through the gage and V is the bridge excitation voltage. From Equation 7 we see that lowering the excitation voltage (or age current) or increasing the gage resistance will decrease the power dissipation. When self-heating may be a problem, higher values of gage resistance should be used.

The temperature rise of the grid is difficult to calculate because many factors influence the heat balance. Unless the gage is submerged in a liquid, most of the heat transfer will occur by conduction to the specimen. Generally, cooling of the gage by convection is undesirable because of the possibility of creating time variant thermal gradients on the gage. These gradients can generate voltages due to the thermocouple effect at the lead wire junctions causing errors in the bridge output voltage. Heat transfer from the gage grid to the specimen is via conduction. The grid surface area and the materials and thicknesses of the carrier and adhesive influence gage temperature. The heat sink characteristics of the specimen are also important.

POWER DENSITY is a parameter used to evaluate a particular gage size and excitation voltage level for a particular application. Power density is the power dissipated by the gage divided by the gage grid area and has units of watts/square inch . Recommended values of power density vary, depending upon accuracy requirements, from 2-10 for good heat sinks such as heavy aluminum or copper sections, to 0.01-0.05 for poor heat sinks such as unfilled plastics. Stacked rosettes create a special problem in that the temperature rise of the bottom gage adds to the two gages above it and that of the center gage adds to the top gage. These may require a very low voltage or different voltages for each of the three gages to have the same temperature at each gage.

Figure 18
Guard wire diverts common mode current away from gage wires



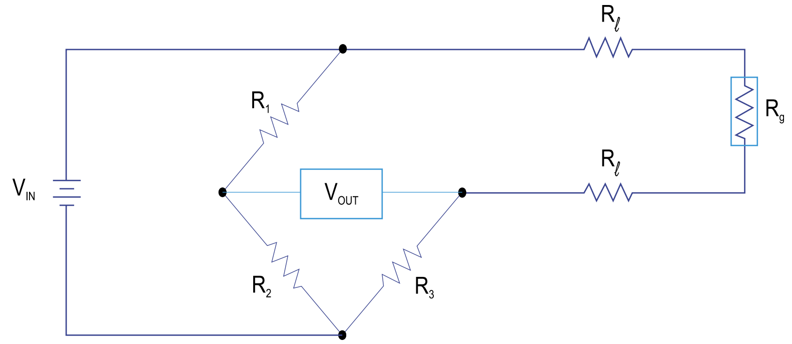


Figure 19
Two-wire 1/4-bridge Connection

One way we can determine the maximum excitation voltage that can be tolerated is by increasing the voltage until a noticeable zero instability occurs. We then reduce the voltage until the zero is once more stable and without a significant offset relative to the zero point at a low voltage. The bridge completion resistors also dissipate power and in practice may be more susceptible to drift from self-heating effects than the strain gage. The stability of the bridge completion resistors is related to load-life, and maintaining only a fraction of rated power in them will give better long-term stability. If the above method of finding the maximum voltage level is used, care should be exercised to insure that the power rating of the completion resistors is not exceeded as the voltage is increased.

Reducing the bridge excitation voltage dramatically reduces gage power, since power is proportional to the square of voltage. However, bridge output voltage is proportional to excitation voltage, so reducing it lowers sensitivity. If the DVM used to read the output voltage has 1 µV resolution, 1 µε resolution can be maintained, with a 1/4-bridge configuration, using a 2 V bridge excitation level. If the DVM has 0.1 µV resolution, the excitation voltage can be lowered to 0.2 V while maintaining the same strain resolution.

Lead Wire Desensitization (Refer to Figure 20)
1/4 and 1/2-bridge, 3-wire connections

AWG	R_g = 120 Ω	R_g = 350Ω
18	0.54%	0.19%
20	0.87%	0.30%
22	1.38%	0.47%
24	2.18%	0.75%
26	3.47%	1.19%
28	5.52%	1.89%
30	8.77%	3.01%

Magnitudes of computed strain values are low by above percent per 100 feet of hard drawn solid copper lead wire at 25 ˚C (77 ˚C)

Table 4

Lead Wire Effects

In the preceding chapter, reference was made to the effects of lead wire resistance on the strain measurement for the various configurations. In a bridge circuit, the lead wire resistance can cause two types of errors. One error is due to resistance changes in the lead wires that are indistinguishable from resistance changes in the gage. The other error is known as LEAD WIRE DESENSITIZATION and becomes significant when the magnitude of the lead wire resistance exceeds 0.1% of the nominal gage resistance. The significance of this source of error is shown in Table 4.

If the resistance of the lead wires is known, the computed values of strain can be corrected for LEADWIRE DESENSITIZATION. In a prior section, we developed equations for strain as a function of the measured voltages for a 1/4-bridge configuration:

$$\frac{\Delta R}{Rg} = \frac{-4Vr}{1+2\,Vr}$$

Equation 5

$$\varepsilon = \frac{-4Vr}{GF\left(1\ +2\,Vr\right)}$$

Equation 6

These equations are based on the assumptions that V_r, is due solely to the change in gage resistance, ΔR_g, and that the total resistance of the arm of the bridge that contained the gage was R_g. Referring to Figure 20 we see that one of the lead wire resistances, R_ℓ, is in series with the gage so the total resistance of that bridge arm is actually $R_g + R_\ell$. If we substitute this into Equation 5 it becomes:

$$\frac{\Delta R}{Rg} = \left(\frac{-4Vr}{1+2\,Vr}\right)\left(\frac{Rg+Rl}{Rg}\right)$$

Equation 9

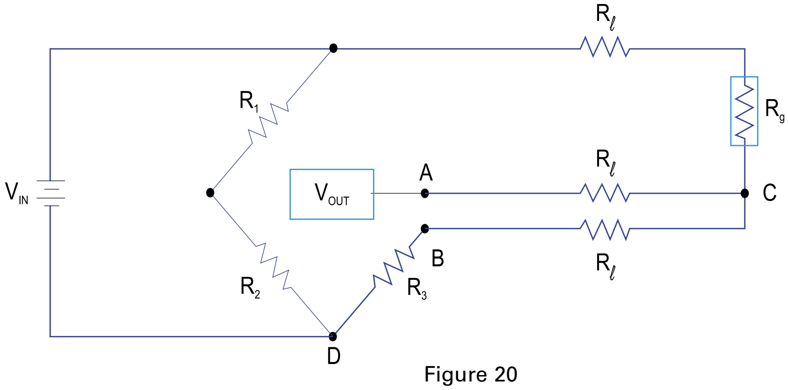


Figure 20
Three-wire 1/4-bridge Connection

Technical Note

Practical Strain Gage

Rewriting the equation for strain, we see that the previous strain equation is in error by a factor of the ratio of the lead wire resistance to the nominal gage resistance.

$$\varepsilon = \frac{-4V_r}{GF\left(1 + 2V_r\right)} \cdot \underbrace{\left(1 + \frac{Rl}{Rg}\right)}_{Error\,Term}$$

Equation 10

This factor is lead wire desensitization and we see from Equation 10 and from Table 4 that the effect is reduced if the lead wire resistance is small and/or the nominal gage resistance is large. If ignoring this term ($1 + R_\ell/R_g$) will cause an unacceptable error, then it should be added to the computer program such that the strains computed with Equation 6 are multiplied by this factor. Appendix B gives the equations for various bridge configurations and the lead wire resistance compensation terms that apply to them. Appendix A has a table containing the resistance, at room temperature, of some commonly used sizes of copper wire.

The most common cause of changes in lead wire resistance is temperature change. The copper used for lead wires has a nominal temperature coefficient of resistance, at 25 ˚C, of 0.00385 Ω/Ω/˚C. For the 2-wire circuit in Figure 19, this effect will cause an error if the temperature during the unstrained reading is different than the temperature during the strained reading. An error occurs because any change in resistance in the gage arm of the bridge during this time is assumed to be due to strain. Also, both lead wire resistances are in series with the gage in the bridge arm, further contributing to the lead wire desensitization error.

The THREE-WIRE method of connecting the gage, shown in Figure 20, is the preferred method of wiring strain gages to a bridge circuit. This method compensates for the effect of temperature on the lead wires. For effective compensation, the lead wires must have approximately the same nominal resistance, the same temperature coefficient of resistance, and be maintained at the same temperature. In practice, this is effected by using the same size and length wires and keeping them physically close together.

Temperature compensation is accomplished because the resistance changes occur equally in adjacent arms of the bridge and therefore the net effect on the output voltage of the bridge is negligible. This technique works equally well for 1/4 and 1/2-bridge configurations. The lead wire desensitization effect is reduced over the two-wire connection because only one lead wire resistance is in series with the gage. The resistance of the signal wire to the DVM doesn't affect the measurement because the current flow in this lead is negligible due to the high input impedance of the DVM. Mathematical correction for lead wire desensitization requires the resistances of the lead wires to be known. The values given in wire tables can be used, but for temperature extremes, measurement of the wires after installation is required for utmost accuracy. Two methods for arriving at the resistance of the lead wires from the instrumentation side of the circuit in Figure 20 follow:

(1) If the three wires are the same size and length, the resistance measured between points A and B, before the wires are connected to the instrumentation, is $2R_\ell$.

(2) Measure the voltage from A-B (which is equivalent to B-C) and the voltage from B-D. Since R_3 is typically a precision resistor whose value is well known. The current in the C-D leg can be computed using Ohm's law. This is the current that flows through the lead resistance so the value of R_ℓ, can be computed, since the voltage from B-C is known. The equation for computing R_ℓ, is:

$$Rl = \frac{VAB}{VBD} \cdot R3$$

Equation 11

Appendix A: Tables

Wire Resistance (Solid Copper wire)

AWG	Ω/ft (25˚C)	Diameter (in.)
18	0.0065	0.040
20	0.0104	0.032
22	0.0165	0.0253
24	0.0262	0.0201
26	0.0416	0.0159
28	0.0662	0.0126
30	0.105	0.010
32	0.167	0.008

Average Properties of Selected Engineering Materials (exact values may vary widely)

Material	Poisson's Ratio, υ	Modulus of Elasticity, E psi x 10^6	Elastic strength (a) Tension (psi)
ABS (unfilled)	-	0.2 - 0.4	4500-7500
Aluminium (2024 -T4)	0.32	10.6	48000
Aluminium (7065 -T6)	0.32	10.4	72000
Red Brass, Soft	0.33	15	15000
Iron-Gray Cast	-	13 - 14	-
Polycarbonate	0.285	0.3 - 0.38	8000-9500
Steel - 1018	0.285	30	32000
Steel - 4130/4340	0.28 - 0.29	30	45000
Steel - 304 SS	0.25	28	35000
Steel - 410 SS	0.27 - 0.29	29	40000
Titanium Alloy	0.34	14	135000

(a) Elastic strength may be represented by proportional limit, yield point, or yield strength at 0.2 percent offset.

Appendix B: Bridge Circuits

Strain Gage Bridge Circuits and Equations

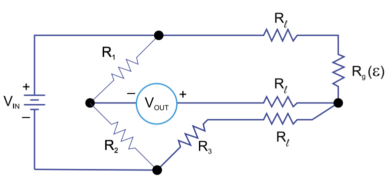
Equations compute strain from unbalanced bridge voltages: sign is correct for V_{in} *and* V_{out} *as shown*

GF = *Gage Factor*: υ = *Poisson's ratio*:

$$V_r = \left[\left(\frac{VOUT}{VIN}\right) strained - \left(\frac{VOUT}{VIN}\right) unstrained\right]$$

ε = *Strain: Multiply by* 10^6 *for micro − strain: tensile is* $(+)$ *and compressive is* $(\_)$

Quarter-bridge Configurations



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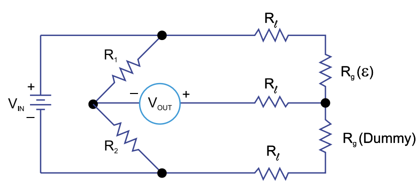
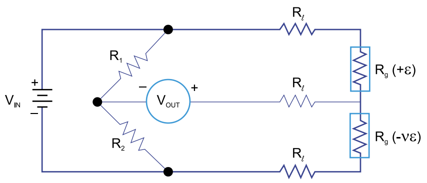


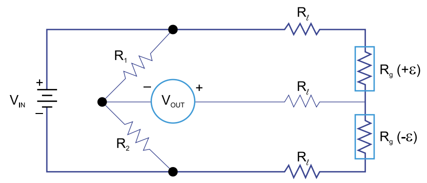
$$\varepsilon = \frac{-4V_r}{GF\left(1 + 2V_r\right)} \cdot \left(1 + \frac{Rl}{Rg}\right)$$

Practical Strain Gage

Appendix B: Bridge Circuits (Continued)

Half-bridge Configurations

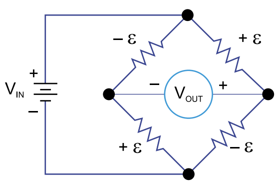


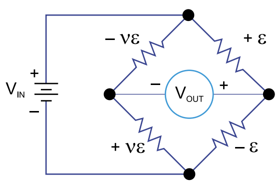


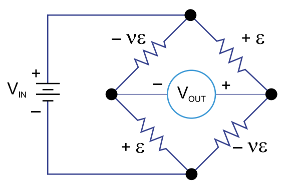
$$\varepsilon = \frac{-4V_r}{GF\left[(1+\upsilon) - 2V_r(\upsilon - 1)\right]} \cdot \left(1 + \frac{R_l}{R_g}\right)$$

$$\varepsilon = \frac{-2V_r}{GF} \cdot \left(1 + \frac{R_l}{R_g}\right)$$

Full-bridge Configurations







$$\varepsilon = \frac{-V_r}{GF}$$

$$\varepsilon = \frac{-2V_r}{GF(\upsilon + 1)}$$

$$\varepsilon = \frac{-2V_r}{GF\left[(\upsilon + 1) - V_r(\upsilon - 1)\right]}$$

Appendix C: Equations

Biaxial Stress State

$$\varepsilon_x = \frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E}$$

$$\varepsilon_z = \upsilon\frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E}$$

$$\sigma_y = \frac{E}{1-\upsilon^2}(\varepsilon_y - \upsilon\varepsilon_x)$$

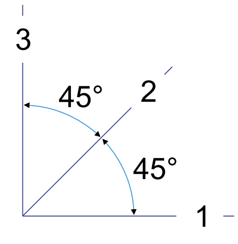
$$\varepsilon_y = \frac{\sigma_y}{E} - \upsilon\frac{\sigma_x}{E}$$

$$\sigma_x = \frac{E}{1-\upsilon^2}(\varepsilon_x + \upsilon\varepsilon_y)$$

$$\sigma_z = 0$$

Rosette equations

Rectangular Rosette

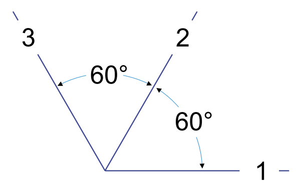


$$\varepsilon_{p,q} = \frac{1}{2}\left[\varepsilon_1 + \varepsilon_3 \pm \sqrt{(\varepsilon_1 - \varepsilon_3)^2 + (2\varepsilon_2 - \varepsilon_1 - \varepsilon_3)}\right]$$

$$\sigma_{p,q} = \frac{E}{2}\left[\frac{\varepsilon_1 + \varepsilon_3}{1-\upsilon} \pm \frac{1}{1+\upsilon}\sqrt{(\varepsilon_1 - \varepsilon_3)^2 + (2\varepsilon_2 - \varepsilon_1 - \varepsilon_3)}\right]$$

$$\theta_{p,q} = \frac{1}{2}TAN^{-1}\frac{2\varepsilon_2 - \varepsilon_1 - \varepsilon_3}{\varepsilon_1 - \varepsilon_3}$$

Delta Rosette



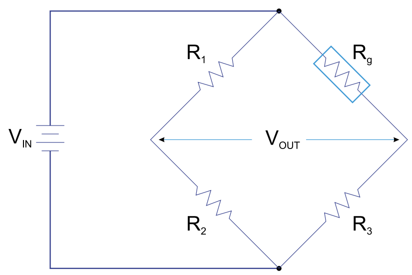
$$\varepsilon_{p,q} = \frac{1}{3}\left[\varepsilon_1 + \varepsilon_2 + \varepsilon_3 \pm \sqrt{2\left[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2\right]}\right]$$

$$\sigma_{p,q} = \frac{E}{3}\left[\frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3}{1-\upsilon} \pm \frac{1}{1+\upsilon}\sqrt{2\left[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2\right]}\right]$$

$$\theta_{p,q} = \frac{1}{2}TAN^{-1}\frac{\sqrt{3}(\varepsilon_2 - \varepsilon_3)}{2\varepsilon_1 - \varepsilon_2 - \varepsilon_3}$$

Appendix D: Instrument Accuracy

1/4-bridge Circuit



Measurement error (in με) due to the instrumentation is often difficult to determine from published specifications. However, accuracy can be computed using the following simplified error expressions. For the 1/4-bridge, add equations 1-6 (N=1). For the 1/2-bridge with two active arms, add equations 2-6 (N=2). For the full bridge with four active arms, add equations 3-6 (N=4).

The total for a measurement must also include gage, lead wire, and if applicable, bridge nonlinearity errors. These are discussed in the body of this application note. Additionally, other equipment imperfections which vary from instrument to instrument must occasionally be considered (e.g., offsets caused by leakage currents due to humidity or ionic contamination on printed circuit boards and connectors).

(1) R_3 change from unstrained to strained reading (due to temperature, load life, etc.)

$$\varepsilon_{error} \approx -\frac{\Delta R3 / R3}{GF}$$

(2) R_1/R_2 change from unstrained to strained reading (due to temperature, load life, etc.)

$$\varepsilon_{error} \approx \frac{\Delta \frac{R1}{R2} \Big/ \frac{R1}{R2}}{GF \cdot N}$$

Digital voltmeters and A/D converters are specified in terms of a ± gain error (% of reading) and a ± offset error (number of counts, in volts). Since strain calculations require two measurements, a repeatable offset error, e.g., due to relay thermal EMF, etc., will cancel, but offset due to noise and drift will not. Assuming that noise and drift dominate, the offset on the two readings will be the root sum of squares of the two offsets. This is incorporated into the formulas.

(3) DVM offset error on bridge measurement

$$\varepsilon_{error} \lessapprox \frac{-4}{V_{IN} \cdot GF \cdot N} \cdot \sqrt{\left(OffsetError_{strained}\right)^2 + \left(OffsetError_{unstrained}\right)^2}$$

Error terms 4-6 can usually be ignored when using high accuracy DVMs (e.g., 5 1/2 digit). These error terms are essentially the product of small bridge imbalance voltages with small gain or offset terms. For equations 4-6, V_{OUT} (the bridge imbalance voltage) is the measured quantity which varies from channel to channel. To calculate worst case performance, the equations use resistor tolerances and measure strain, eliminating the need for an exact knowledge of V_{OUT}.

(4) DVM gain error on bridge measurement

$$\varepsilon_{error} \approx \frac{-4}{V_{IN} \cdot GF \cdot N} \cdot \left[\left(V_{out}\right) \cdot \left(Gain\ Error\right)_{strained\ reading} - \left(V_{out}\right) \cdot \left(Gain\ Error\right)_{unstrained\ reading}\right]$$

$$\lessapprox -\varepsilon_{measured} \cdot \left(Gain\ Error\right)_{strained\ reading} - \frac{\Sigma\ tolerances\ on\ R1/R2, R3, R_g}{GF \cdot N} \cdot \left(\begin{matrix} Gain\ Error\ change \\ strained - unstrained \end{matrix}\right)$$

The bridge excitation supply can be monitored with a DVM or preset using a DVM, and allowed to drift. In the first case, supply related errors are due only to DVM gain and offset terms, assuming a quiet supply. In the second case, since power supply accuracy is usually specified in terms of a ± gain and a ± offset from the initial setting, identical equations can be used. Also for the second case, note that the strained reading gain error is the sum of the DVM and excitation supply gain errors, while the strained reading offset error is the root sum of squares of the DVM and excitation supply offset errors.

Practical Strain Gage

(5) Offset error on supply measurement (or on supply drift)

$$\varepsilon_{error} \approx \frac{-4}{VIN \cdot GF \cdot N} \cdot \left[(V_{out}) \cdot (Offset\ Error)_{strained\ reading} - (V_{out}) \cdot (Offset\ Error)_{unstrained\ reading} \right]$$

$$\lessapprox \frac{\varepsilon_{measured}}{VIN} \cdot (Offset\ Error)_{strained\ reading} - \frac{\Sigma\ tolerances\ on\ R_1/R_2, R_3, R_g}{VIN \cdot GF \cdot N} \cdot \sqrt{(OffsetError_{strained})^2 + (OffsetError_{unstrained})^2}$$

(6) Gain error on supply measurement (or on supply drift)

$$\varepsilon_{error} \approx \frac{-4}{VIN \cdot GF \cdot N} \cdot \left[(V_{out}) \cdot (Gain\ Error)_{strained\ reading} - (V_{out}) \cdot (Gain\ Error)_{unstrained\ reading} \right]$$

$$\lessapprox \varepsilon_{measured} \cdot (Gain\ Error)_{strained\ reading} + \frac{\Sigma\ tolerances\ on\ R_1/R_2, R_3, R_g}{GF \cdot N} \cdot \left(\begin{matrix} Gain\ Error\ change \\ strained - unstrained \end{matrix} \right)$$

Example

Evaluate the error for a 24 hour strain measurement with a ± 5 ˚C instrumentation temperature variation. This includes the DVM and the bridge completion resistors but not the gages. The hermetically sealed resistors have a maximum TCR of ±3.1 ppm/˚C, and have a ±0.1% tolerance. The DVM/scanner combination, over this time and temperature span. has a 0.004% gain error and a 4 μV offset error on the 0.1 V range where the bridge output voltage, V_{OUT}, will be measured. The excitation supply is to be set at 5 V using the DVM. The DVM has a 0.002% gain error and a 100 μV off-set error on the 10 V range. Over the given time and temperature span, the supply has a 0.015% gain error and a 150 μV offset error and will not be remeasured. The mounted gage resistance tolerance is assumed to be ±0.5% or better. The strain to be measured is 3000 με and the gage factor is assumed to be +2.

Equation	1/4-bridge	1/2-bridge	Full Bridge
(1) R_3	15.5	-	-
(2) R_1/R_2	31.0	15.5	-
(3) V_{out} offset	2.3	1.1	0.6
(4) V_{out} gain	0.4	0.4	0.3
(5) V_{in} offset	0.3	0.2	0.2
(6) V_{in} gain	1.3	1.1	1.0
Sum	±50.8 με	±18.3 με	±2.1 με

Notice that the temperature, as given, can change by as much as ± l0 ˚C between the unstrained and strained measurements. This is the temperature change that must be used to evaluate the resistor changes due to TCR. The R1/R2 ratio has the tolerance and TCR of two resistors included in its specification so the ratio tolerance is ±0.2% and the ratio TCR is ±6.2 ppm/˚C. The gain error change on the bridge output measurement, and on the excitation measurement, can be as much as twice the gain error specification. The table above shows the total error and the contribution of the individual error equations 1-6.

Conclusions

Based upon this example, several important conclusions can be drawn:

- Surprisingly large errors can result even when using state-of-the-art bridge completion resistors and measuring equipment.
- Although typical measurements will have a smaller error, the numbers computed reflect the guaranteed instrumentation performance.
- Measuring the excitation supply for both the unstrained and strained readings not only results in smaller errors but allows the use of an inexpensive supply.
- Bridge completion resistor drift limits quarter and half-bridge performance.
- Changes due to temperature, moisture absorption, and load life, require the use of ultra-stable hermetically sealed resistors.